Comment on "Validity of certain soft-photon amplitudes"

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Abstract

The criteria suggested by Welsh and Fearing to judge the validity of certain soft-photon amplitudes are examined. We comment on aspects of their analysis which lead to incorrect conclusions about published amplitudes and point out important criteria which were omitted from their analysis.

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In a recent paper [1] Welsh and Fearing discussed the validity of certain soft-photon amplitudes. Their study focused on two issues, a phase space question and a symmetrization problem. We comment on aspects of their arguments and procedures which lead to improper conclusions.

Evidence suggests that at least two different classes of soft-photon amplitudes are required to describe nuclear bremsstrahlung processes. The two-u-two-t special (TuTts) amplitudes, which represent a class of amplitudes evaluated using the Mandelstam variables (u_1, u_2, t_p, t_q), were found to be optimal for processes involving strong u-channel exchange effects [2–4]. The two-s-two-t special (TsTts) amplitudes, which represent a class of amplitudes evaluated at (s_i, s_f, t_p, t_q), were found to be optimal for processes involving strong s-channel resonance effects [2,5,6]. (The notation is defined in Ref. [4].) These two classes of amplitudes can be derived for two bremsstrahlung cases: (*i*) $A + A \to A + A + \gamma$ and (*ii*) $A + B \to A + B + \gamma$. Alternative derivations were discussed in Ref. [2,4], but neither TsTts nor TuTts amplitude has ever been defined for the third case (*iii*) $A + B \to C + D + \gamma$. (Here A, B, C, and D represent particles having different masses and charges.) The amplitudes given by Eqs. (8), (9), and (11) of Ref. [1] cannot be rigorously "derived" for the third case (*iii*); they were improperly "generalized" by the authors. The danger of postulating rather than deriving results is further illustrated in Eq. (31) of Ref. [1], where three invalid expressions for Δ_i are given.

The phase space problem related to the TsTts amplitude has already been discussed in Ref. [6]. Because the center-of-mass angle θ_{cm} cannot be defined for some kinematical points involving (s_f, t_p) and (s_f, t_q), which correspond to those points that lie outside the measurable region shown in Fig. 2 of Ref. [1], the two-energy-four-angle version of the TsTts amplitude cannot be used. To circumvent this problem in Ref. [3], the special two-energy-two-angle (TETAS) amplitudes proposed in Ref. [6] were utilized for all TsTts calculations.

Because TuTts and TETAS amplitudes effectively describe different bremsstrahlung processes, the theoretical constraints to be imposed upon them can differ. For example, the TETAS amplitudes were shown to be the most successful in describing bremsstrahlung pro-

cesses near a resonance [5–9]. They satisfy an important criterion which was neglected in Ref. [1]. That is, in order to describe a bremsstrahlung process associated with a significant resonance, a valid amplitude must predict the correct (energy) position and width of the resonant peak, as observed in the bremsstrahlung spectrum. Using Eqs. (24) and (25) of Ref. [5], this criterion was investigated thoroughly [5,6,9]. Processes like $\pi^+ p\gamma$ [7,8] and $p^{12}C\gamma$ [9] in the region of a resonance can only be well described by amplitudes which are evaluated at s_i and s_f; the TETAS amplitudes were demonstrated to provide an excellent description of those processes. The conventional Low amplitude fails to describe the $\pi^\pm p\gamma$ and $p^{12}C\gamma$ data in the vicinity of a resonance; in particular, it predicts incorrectly the position and width of the resonance peaks observed in the $p^{12}C\gamma$ spectrum [5,6,9].

In Ref. [1] it was concluded that "the TuTts amplitude cannot be antisymmetrized while being written in terms of the measurable *pp* elastic amplitude". Furthermore, it was stated that "the failure in symmetrization arises at O(K/K)". We point out that this statement and conclusion are incorrect. Firstly, the TuTts amplitude M_μ^{TuTts} ($\equiv M_{1\mu}^{TuTts}$) given by Eq. (31) in Ref. [4] *is* properly antisymmetrized, and it has been expressed in terms of the standard GGMW representation [10] for the *pp* elastic process. This amplitude satisfies the Pauli principle and other theoretical constraints. Secondly, as shown in Ref. [4], another TuTts amplitude $M_{2\mu}^{TuTts}$ [Eq. (49)] can be obtained if the gauge invariance condition alone (without imposition of the Pauli principle) is used in the derivation. This amplitude, which was used in Ref. [3], is not the unique representative of the TuTts class. The fact that this particular amplitude fails to satisfy the Pauli principle does not imply that the entire class of TuTts amplitudes must violate the Pauli principle. Finally, the amplitude $M_{2\mu}^{TuTts}$ violates the Pauli principle at order O(K), not at O(K/K) as stated in Ref. [1]. This can be seen by direct examination of the internal contribution to the amplitude. Alternatively, if one wishes to exhibit this violation by comparing with the amplitude satisfying the Pauli principle, then the comparison should be made with M_μ^{TuTts} and not the incorrect amplitude $M_{TuTts}^{S\mu}$ given by Eq. (27) [or by subtracting Eq. (44) from Eq. (38)] of Ref. [1]. The same answer is obtained from either procedure. That the violation is O(K) suggests that the

two amplitudes M_{μ}^{TuTts} and $M_{2\mu}^{TuTts}$ should predict very similar $pp\gamma$ cross sections except for kinematic conditions in which both proton scattering angles are very small and the photon angle ψ_{γ} is near 180^{o}, which is borne out by numerical calculations. As shown in Fig. 1, the coplanar $pp\gamma$ cross section at 157 MeV and for small symmetric proton scattering angles of 10^{o} are almost the same for the two amplitudes except for ψ_{γ} near 180^{o}. It is also worth noting that the TRIUMF cross section data at 280 MeV, which disagree with some predictions calculated using the Low amplitude [11,3], can actually be described satisfactorily by the M_{μ}^{TuTts} amplitude.

The incorrect conclusion drawn in Ref. [1] follows directly from the procedures employed in constructing a soft-photon amplitude. These procedures, which differ from those utilized in Ref. [2] and [4], involve three steps: (A) Choose an unsymmetrized bremsstrahlung amplitude M_{μ} which includes both the external and internal contributions. (B) Obtain an exchange amplitude $(M_{\mu})_{p_f \leftrightarrow qf}$ by interchanging p_f with q_f (the momenta of the two outgoing protons) in M_{μ}. (C) Construct the symmetrized amplitude as

$$M^{S(A)} = M_{\mu} \pm (M_{\mu})_{p_f \leftrightarrow q_f},$$

where the $S(A)$ corresponds to the $+(-)$ sign and describes two spin-0 (spin-$\frac{1}{2}$) particles. Obviously, the initial step (A) is the crucial one. The exchange amplitude $(M_{\mu})_{p_f \leftrightarrow qf}$ can always be obtained from a given M_{μ}, even when M_{μ} is not valid. That is, a wrong amplitude M_{μ} leads to a wrong amplitude $M^{S(A)}$. Step (A) specifies no detailed prescription for constructing M_{μ}. Without a guiding prescription, an incorrect expression for M_{μ} can be chosen. The treatment of the TuTts case in Ref. [1] is a prime example. Furthermore, Welsh and Fearing have emphasized that any soft-photon amplitude constructed should be written in terms of the corresponding elastic amplitude. Certainly that is correct, but this condition is automatically satisfied if the procedures outlined in Ref. [4] are followed. (A properly antisymmetrized GGMW amplitude is the input for generating the $pp\gamma$ amplitude.) However, a violation of this condition may arise when the procedures of Ref. [1] are employed.

The procedures advocated in Ref. [1] can be used to obtain the correct amplitude M_{μ}^{TuTts},

provided that the procedures introduced in Ref. [4] are utilized to generate a correct expression for M_μ. That is, M_μ should depend upon V_μ {Eq. (32) of Ref. [4]} and not $\bar{V}_\mu$ {Eq. (43) of Ref. [4]}. In addition, the constraint given by Eq. (7) of Ref. [4] must be imposed in the derivation.

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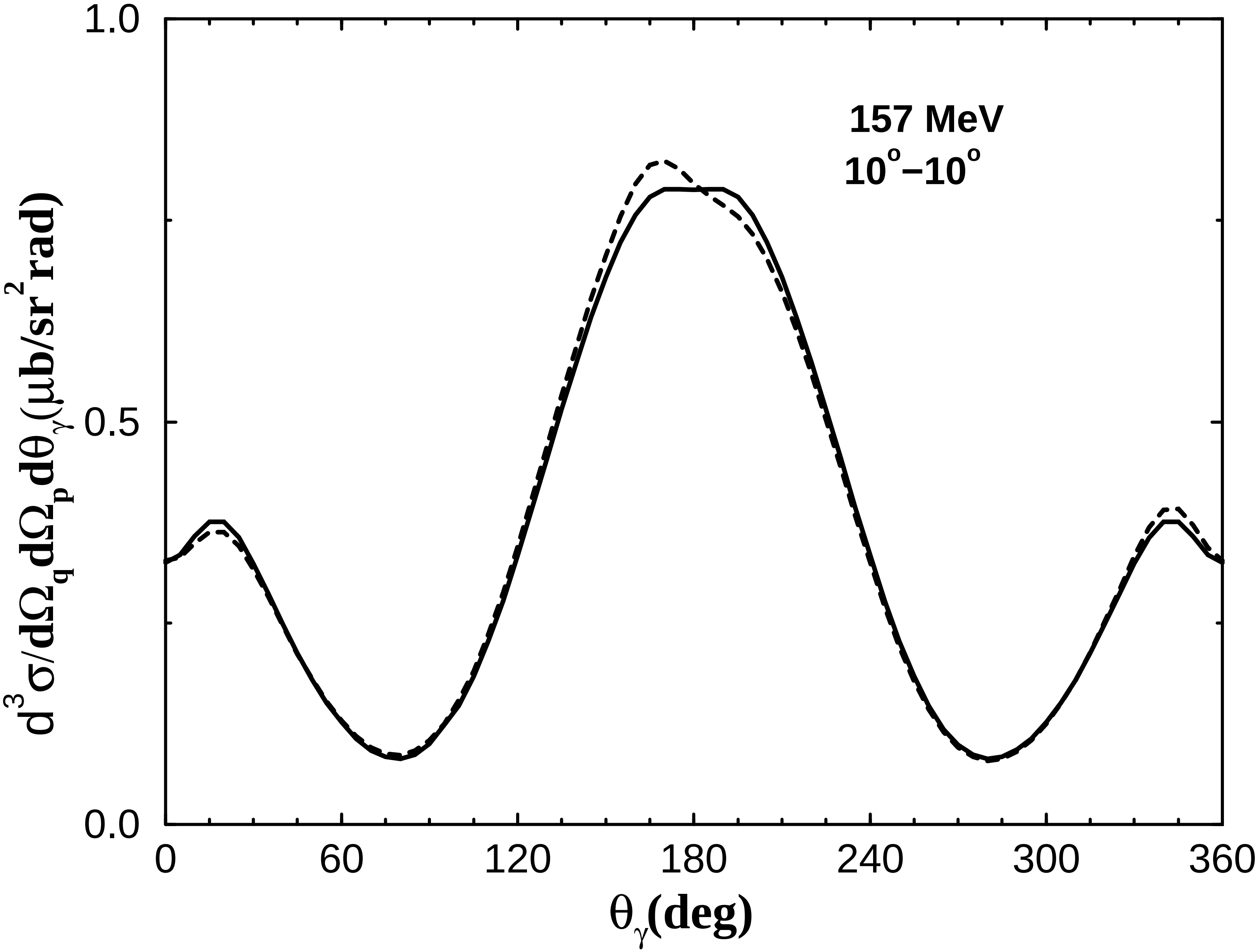
Fig. 1. Coplanar $pp\gamma$ cross section at 157 MeV for symmetric 10^o proton angles. The solid (symmetric) and dashed curves were calculated using the amplitudes M_{μ}^{TuTts} and $M_{2\mu}^{TuTts}$, respectively.


157 MeV
10°–10°
1.0
0.5
0.0
0
60
120
180
240
300
360
$d^3\sigma/d\Omega_q d\Omega_p d\theta_\gamma$ (μb/sr^{2}rad)
θ_γ(deg)